

# Cooperatives and Area Yield Insurance: A Theoretical Analysis

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## Abstract

The purpose of this paper is to theoretically investigate the potential benefits that arise from a cooperative selling a government subsidized area-yield contract (i.e., the Group Risk Plan). The indemnities in area-yield contracts are triggered by a geographically determined yield (e.g., a county-wide yield average) instead of the more conventional individual actual production history. Therefore, an area-yield contract would be appropriate for managing the cooperative's systemic throughput risk. The cooperative would also capture some of the substantial government subsidies that are normally given to a private insurance company. Our primary finding is that farmers should be indifferent when considering the decision to purchase area-yield insurance from a private company or encompass that business in their cooperative. We derive this result for the specific case of costless insurance and assume a Pareto Optimal contract. Under these assumptions, the government subsidies that the cooperative would hope to capture are simply a net deduction in their premiums. In other words, the benefit they capture from the subsidies is the same when they purchase the insurance from an outside firm or internally.

# 1 Introduction

Agricultural cooperatives exist, in part, because they are a risk management strategy for their members (Sporleder and Goldsmith; Schrader-class; Zeuli). The cooperative can help mitigate farm price risk by storing, pooling, and selling raw commodities under contract or by processing the commodities and selling retail products that offer more stable prices than commodities. Since cooperatives aggregate farm output, an individual's yield loss is offset by the other members. This system helps ensure that the cooperative can always meet the terms of any supply contracts. However, it also exposes the cooperative and therefore the members (the residual claimants) to throughput and profit uncertainty. This uncertainty is a function of each member's yield variability.<sup>1</sup> Therefore, the cooperative faces both independent and systemic yield risk.

Cooperatives can implement several strategies to help manage their throughput risk, the variability of the firm's supply of input, although none is without some distinct disadvantage (Zeuli). Some cooperatives have promoted hedge-to-arrive contracts, which guarantee that each member will deliver a certain volume of grain on a given date (Baumel and Lasley). New generation cooperatives solve this problem by shifting the risk to the members. Farmers enter into quantity rather than yield-based contracts and must purchase the difference if they experience any shortfalls. Therefore, although the cooperative is ensured against throughput risk, the farmer loses one of the benefits of the cooperative form of business. Some cooperatives have considered offering various yield insurance products to their members (Black, Barnett, and Hu).

The purpose of this paper is to theoretically investigate the potential benefits that arise from a cooperative selling a government subsidized area-yield contract (i.e., the Group Risk Plan). The indemnities in area-yield contracts are triggered by a geographically determined yield (e.g., a county-wide yield average) instead of the more conventional individual actual production history (Skees, Black, and Barnett). Therefore, an area-yield contract would be appropriate for managing the cooperative's systemic throughput risk. The cooperative would also capture some of the substantial government subsidies that are normally given to a private insurance company. The USDA's Risk Management Agency provides three types of subsidies (Young, Vandever, and Schnepf): they discount the premiums to make the product more affordable for farmers; they pay the insurance providers a fixed percentage of gross premiums to cover their administrative and delivery expenses; and they subsidize the insurance providers' underwriting risk. Cooperatives may be able to deliver the insurance at lower cost and pass the administrative cost-savings onto their members (Black, Barnett, and Hu). Black, Barnett and Hu provide an empirical investigation into a particular case of

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<sup>1</sup>The cooperative purchases as much of its input from its members as possible although they may purchase product on the open market when faced with shortfalls. This act exposes them to price and quantity uncertainty.

the cooperative providing the insurance, but no theory. Mahul (2001) shows theoretically that insurance programs based on producer’s individual yields have been disappointing due to the fact that premiums have not been sufficient to cover the total cost of insurance. One alternative to overcome this problem is to replace individual insurance with an area yield insurance system that strongly reduces both moral hazard problems and insurance’s cost.

We compare two insurance provision scenarios using a standard utility maximizing framework with a representative farmer agent. In the first model, an area-yield crop insurance product is sold by a risk neutral agent or privately owned firm (the insured are not owners). In the second model, the same crop insurance product is sold jointly by a risk neutral insurer and a cooperative (owned by the insured). In both models we find Pareto Optimal insurance contracts when insurance is costless (i.e., the administrative and underwriting costs are zero). We compare the welfare of a representative farmer under both scenarios and also provide some qualitative extensions that consider heterogeneous agents.

The paper proceeds as follows. We discuss the context and provide the basic model, where the area yield insurance is provided by a private company, in section 1. This is followed by the alternative model exposition, where the insurance is provided jointly by the cooperative and a private risk neutral insurer. In section 3 we compare the optimal indemnities under each case and in section 4 we compare the welfare effects. We conclude the paper by summarizing our findings and suggesting areas for future research.

## 2 Context and Basic Model (a Private Insurance Provider)

Consider an environment in which there are  $n$  farmers ( $i = 1, 2, \dots, n$ ) in some geographically distinct crop production region who produce the same commodity. The commodity is characterized by stochastic volume-based (e.g., bushels) yields  $y_i$  and is sold at a unit price  $p$ . Since all the farmers face the same price, we normalize the commodity price such that  $p = 1$ . To compare parallel situations, we assume that all farmers are members of a cooperative that processes their crop; in this basic model the cooperative has no relationship with the insurance provider. The cooperative’s annual profits  $\pi^f(y)$  are a function of the aggregate yield  $y$  of the region. The cooperative’s annual profits are specified as follows:

$$\pi^f(y) = p^f F(y) - C(y)$$

where  $F$  is a production function that produces a single final product,  $p^f$  is the price of that final good, and  $C(y) \geq 0$  represents the cost of production. We assume that  $F$ ,  $p^f$ , and  $C$  are such that in a relevant domain  $D \subset \mathbb{R}_+$ ,  $\pi^f(y)$  is a twice continuously differentiable function of  $y$ , increasing and concave. For example,  $F(y) = y^\alpha$ ,  $\alpha \in (0, 1)$ ;  $p^f = 1$ ,  $C = 0$  and  $D = \mathbb{R}_+$ . We let  $\pi_i(y) = \theta_i \pi^f(y)$  represent the individual farmer  $i$ ’s share of the cooperative’s profits, where  $\theta_i \in [0, 1]$ .

Following Mahul (1999), and without loss of generality, we project the stochastic individual yield  $y_i$  onto the stochastic area yield  $y$  to obtain the following decomposition

$$y_i = \mu_i + \beta_i(y - \mu) + \varepsilon_i \quad (1)$$

where

$$\begin{aligned} \beta_i &= \text{cov}(y_i, y) / \text{var}(y) \\ E\varepsilon_i &= 0; \quad \text{cov}(\varepsilon_i, y) = 0 \\ Ey_i &= \mu_i; \quad Ey = \mu. \end{aligned}$$

To manage their risk exposure, each farmer may buy area yield insurance coverage from a private insurance provider (the farmers have no ownership stake in the insurance company); the farmer will choose an indemnity level  $I(y)$  based on the aggregate yield and be charged a corresponding premium  $P$ . A Pareto Optimal insurance contract is characterized by a premium  $P^*$  and an indemnity level,  $I(y)^*$ , that maximizes the farmer's expected utility function subject to a participation constraint for the insurance provider. Stated more formally, the Pareto Optimal contract solves the following problem:

$$\max_{P, I(\cdot)} Eu(W_i) \quad (2)$$

$$\begin{aligned} \text{s.t.} \quad py_i + I(y) + \pi_i(y) - P &= W_i \\ 0 &\leq I(y) \\ V_0 &\leq V(P - Ec(I(y)) + T) \end{aligned} \quad (3)$$

where  $W_i$  represents the wealth of the farmer. We let  $V$  denote the insurance provider's utility function,  $V_0$  is his reservation utility value,  $c$  is an increasing linear function measuring the total administration and underwriting costs of providing the insurance ( $c \geq 1$ ) and  $T$  corresponds to an exogenous government transfer that would offset these costs (i.e., government subsidies on administration and underwriting costs). We assume that the insurance provider is risk neutral,  $V' = 1$ .

At the optimum the participation constraint is binding so the problem can be reduced as follows:

$$\max_{P, I(\cdot)} Eu(y_i + I(y) + \pi_i(y) - P) \quad (4)$$

$$\begin{aligned} \text{s.t.} \quad 0 &\leq I(y) \\ V_0 + Ec(I(y)) &= P + T \end{aligned} \quad (5)$$

Substituting (1) we can define the problem more completely as

$$\max_{P, I(\cdot)} Eu(\mu_i + \beta_i(y - \mu) + \varepsilon_i + I(y) + \pi_i(y) - P) \quad (6)$$

$$\begin{aligned} \text{s.t. } 0 &\leq I(y) \\ V_0 + Ec(I(y)) &= P + T \end{aligned} \quad (7)$$

The solution of this problem is given in the following proposition.

**Proposition 1** *The Pareto Optimal insurance contract that solves problem (6) satisfies the following conditions:*

$$\begin{aligned} I(y) &= \max\{\beta_i(y^c - y) + (\pi_i(y^c) - \pi_i(y)), 0\} \quad \text{if } \beta_i > 0 \\ I(y) &= \max\{-\beta_i(y - y^{c'}) - (\pi(y) - \pi(y^{c'})), 0\} \quad \text{if } \beta_i < 0 \\ P &= V_0 + Ec(I(y)) - T \end{aligned}$$

where  $y^c, y^{c'}$  are called critical yields and defined by

$$\begin{aligned} I(y^c) &= 0 \quad \text{if } \beta_i > 0 \\ I(y^{c'}) &= 0 \quad \text{if } \beta_i < 0 \end{aligned}$$

where we have assumed that

$$\lim_{x \rightarrow 0} \pi_i(x) < |\beta_i| \quad \text{if } \beta_i < 0 \quad (8)$$

**Proof.** See the appendix.<sup>2</sup> ■

Some insights about the characterization of critical yields are provided in the next proposition.

**Proposition 2** *When the insurance is costless, which means  $c = 1$ , the critical yields are given by the upper and lower bounds of the aggregate yield:*

$$\begin{aligned} y^c &= y^{\max} \quad \text{if } \beta_i > 0 \\ y^{c'} &= y^{\min} \quad \text{if } \beta_i < 0 \end{aligned}$$

*If the insurance is costly, which means  $c > 1$ , the critical yields do not reach the bounds of the aggregate yield.*

**Proof.** See the appendix. ■

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<sup>2</sup>Please contact the authors for a copy of the appendix containing all proofs.

## 2.1 A Cooperative is the Insurance Provider

In this section we consider an alternative scenario, one in which the cooperative jointly with a risk neutral agent (instead of a private insurance provider) sells area-yield insurance to its members. We assume that the two business divisions are integrated without cost or additional benefits to the cooperative. The cooperative's total profit function  $\pi$  is specified as follows:

$$\pi = \pi^f(y) + \theta(\pi^S(\tilde{I}(y), \tilde{P}) + T) \quad (9)$$

where  $\pi^f(y)$  represents profits obtained from the agricultural enterprise (processing) specified above,  $\pi^S(\tilde{I}(y), \tilde{P})$  represents profits obtained from selling insurance to members and  $\theta$  is the participation of the cooperative in the insurance firm. The profits from selling insurance are a function of the aggregate premiums and indemnities purchased by all members:

$$\pi^S(\tilde{I}(y), \tilde{P}) = \sum_{i=1}^n (P_i - cI_i(y)).$$

A Pareto Optimal insurance contract in this scenario is characterized by a premium  $P^*$ , an indemnity level,  $I(y)^*$ , and a share of cooperative profits  $\theta^*$  that maximizes the farmer's expected utility function subject to a participation constraint. Stated more formally, the Pareto Optimal contract solves the following problem:

$$\max_{P, I(\cdot)} Eu(y_i + I(y) + \pi_i(y) - P + \theta(P - I(y) + T)) \quad (10)$$

$$\begin{aligned} \text{s.t.} \quad & 0 \leq I(y) \\ & V_0 = (1 - \theta_i)(P - Ec(I(y)) + T) \end{aligned} \quad (11)$$

We let  $V$  denote the utility function of a risk-neutral investor and  $V_0$  his reservation utility value. This specification, which is standard in the insurance literature, is required because we are dealing with a representative agent. Intuitively, if we consider a group of heterogeneous farmers in the cooperative (i.e., characterized by different  $\beta$ 's) they could theoretically share each other's risk. It is possible, however<sup>3</sup> that their welfare improvement over the case where they purchase it from the private insurer would be minimal. However, if they were also characterized by different utility functions, then their welfare gains might increase.

For ease of exposition, we drop the subscript  $i$  in the remainder of our analysis. The following proposition establishes the Pareto Optimal contract.

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<sup>3</sup>A natural extension of this paper would include this heterogeneous case.

**Proposition 3** *If the insurance is costless and the assumption (8) holds, then the Pareto Optimal insurance contract that solves problem (10) satisfies:*

$$\begin{aligned}
(1 - \theta)I(y) &= \max\{\beta_i(y^c - y) + (\pi_i(y^c) - \pi_i(y)), 0\} \quad \text{if } \beta_i > 0 \\
(1 - \theta)I(y) &= \max\{-\beta_i(y - y^{c'}) - (\pi(y) - \pi(y^{c'})), 0\} \quad \text{if } \beta_i < 0 \\
(1 - \theta)P &= (1 - \theta)Ec(I(y)) - (1 - \theta)T + V_0 \\
\theta &\in [0, 1)
\end{aligned}$$

where  $y^c, y^{c'}$ , the critical yields, are defined by

$$\begin{aligned}
I(y^c) &= 0 \quad \text{if } \beta_i > 0 \\
I(y^{c'}) &= 0 \quad \text{if } \beta_i < 0
\end{aligned}$$

**Proof.** See the appendix. ■

### 3 Comparing Optimal Indemnities

For the sake of brevity, we restrict our comparison to the case in which the insurance is costless and  $\beta > 0$ . Given these conditions, we know that the critical yield is as follows:

$$y^c = y^{\max} \quad \text{for all } \theta \in [0, 1)$$

We denote our base case, when the insurance provider is a private firm, with the superscript 0, and the alternative, when the cooperative jointly sells the insurance with a risk neutral agent, with the superscript 1. The indemnity schedule for the representative farmer can be specified as follows:

$$\begin{aligned}
I_1(y) &= \frac{\beta(y^{\max} - y)}{(1 - \theta)} + \frac{(\pi^f(y^{\max}) - \pi^f(y))}{(1 - \theta)} \\
I_0(y) &= \beta(y^{\max} - y) + (\pi^f(y^{\max}) - \pi^f(y))
\end{aligned}$$

and the connection between the two indemnities is

$$(1 - \theta)I_1(y) = I_0(y)$$

therefore

$$I_1(y) - I_0(y) = \theta I_1(y) \geq 0; \quad y \in [0, y^{\max}].$$

Given these conditions, we are able to establish the following result.

**Proposition 4** a) When insurance is costless and  $\beta > 0$ , the indemnity schedule for a representative farmer when the cooperative jointly sells the insurance with a risk neutral agent is uniformly equal to or greater than when the insurance is sold by a private company.

b) Under the same previous assumptions and when the cooperative runs jointly the insurance with a risk neutral agent the indemnity schedule is an increasing function of  $\theta$  for all  $y \in [0, y^{\max}]$ .

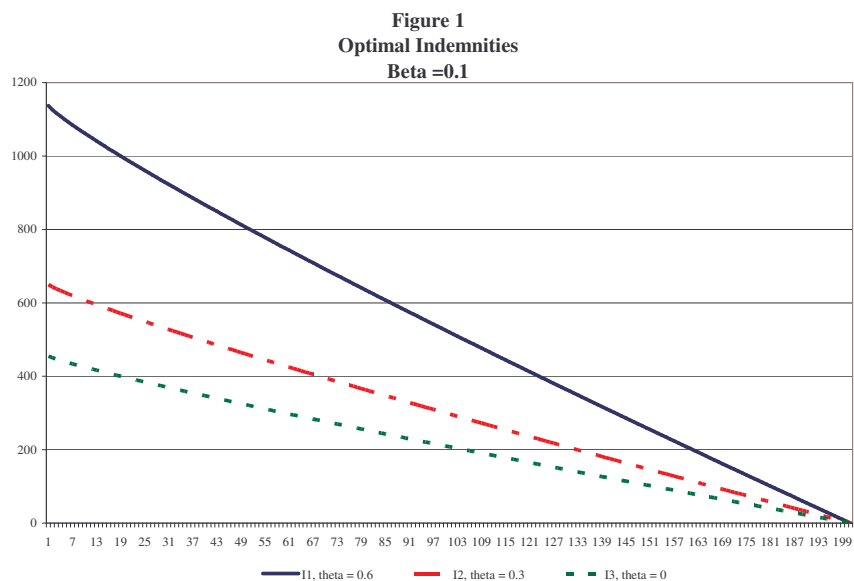
**Proof.** We already showed that

$$I_1(y) - I_0(y) = \theta I_1(y) \geq 0; \quad y \in [0, y^{\max}].$$

therefore part a) is demonstrated.

Part b) follows by simply noting that  $g_1(\theta) = \frac{\theta}{1-\theta}$  is increasing in  $\theta$  for all  $\theta \in (0, 1)$ . ■

According to these propositions, when the cooperative runs the insurance jointly with a risk neutral agent, the farmer loses his ability to share the risk he is facing. In other words, he would be facing more risk and according to that he requires a higher indemnity to offset this higher exposure to risk. The same analysis applies when comparing two scenarios with different profits share ( $\theta$ ). Higher  $\theta$  means more risk exposure and hence a higher indemnity is required. The scenario is illustrated in figure 1.



## 4 Welfare Analysis

First we would like to compare expected utilities under both scenarios. That is to say we want to compare  $Eu(W_0)$  to  $Eu(W_1)$  where we keep using subindex 0 to denote the competitive case and subindex 1 to denote the case in which the cooperative is the insurer.  $W_j$  denotes wealth when analyzing case  $j = 1, 2$ . The next proposition establishes the welfare neutrality for the farmer when the cooperative runs the insurance in the case that the insurance is costless.

**Proposition 5** *If the insurance is costless then the farmer is indifferent about the scenarios in which the insurance is jointly ran by the cooperative and a risk neutral insurer or it is ran by a risk neutral insurer alone.*

**Proof.** *When the insurance is costless we have that  $y^c = y_{\max}$ . In this case we have for  $j = 0, 1$ ,*

$$Eu(W_j) = \int_y \int_{\epsilon} u(W_j) f(y, \epsilon) dy d\epsilon$$

*We have that  $I_j(y) > 0$  for all  $0 < y < y_{\max}$ ,  $j = 0, 1$ . Therefore, if  $\beta > 0$  we have*

$$\begin{aligned} u(W_0) &= u(\mu + \beta(y - \mu) + \varepsilon + I_0(y) + \pi(y) - P_0) \\ &= u(\mu + \beta(y - \mu) + \varepsilon + \beta(y^c - y) + (\pi^f(y^c) - \pi^f(y)) + \pi^f(y) - P_0) \\ &= u(\mu + \varepsilon + \beta(y^c - \mu) + \pi^f(y^c) - P_0) \end{aligned}$$

*but*

$$P_0 = V_0 + E(I_0(y)) - T$$

*and*

$$E(I_0(y)) = \beta(y^c - \mu) + \pi^f(y^c) - E\pi^f(y)$$

*Therefore we have*

$$\begin{aligned} Eu(W_0) &= \widehat{u}(\mu + \beta(y_{\max} - \mu) + \pi^f(y_{\max}) - V_0 - E(I_0(y)) + T) \\ &= \widehat{u}(\mu + \beta(y_{\max} - \mu) + \pi^f(y_{\max}) - V_0 - \beta(y_{\max} - \mu) - \pi^f(y_{\max}) + E\pi^f(y) + T) \\ &= \widehat{u}(\mu - V_0 + E\pi^f(y) + T) \end{aligned}$$

*and the same result holds true for the case  $\beta < 0$ . Similarly for  $i = 1$  we have*

$$\begin{aligned} u(W_1) &= u(\mu + \beta(y - \mu) + \varepsilon + I_1(y) + \pi^f(y) - P_1 + \theta(P_1 + T - I_1(y))) \\ &= u(\mu + \beta(y - \mu) + \varepsilon + (1 - \theta)I_1(y) - (1 - \theta)P_1 + \pi^f(y) + \theta T) \end{aligned}$$

*but if  $\beta > 0$*

$$(1 - \theta)I_1(y) = \beta(y^c - y) + (\pi^f(y^c) - \pi^f(y))$$

so

$$\begin{aligned} u(W_1) &= u(\mu + \beta(y - \mu) + \varepsilon + \beta(y^c - y) + (\pi^f(y^c) - \pi^f(y)) - (1 - \theta)P_1 + \pi^f(y) + \theta T) \\ &= u(\mu + \beta(y^c - \mu) + \varepsilon + \pi^f(y^c) - (1 - \theta)P_1 + \theta T) \end{aligned}$$

but

$$(1 - \theta)P_1 = (1 - \theta)E(I_1(y)) - (1 - \theta)T + V_0$$

and

$$(1 - \theta)E(I_1(y)) = \beta(y^c - \mu) + (\pi^f(y^c) - E\pi^f(y))$$

therefore

$$\begin{aligned} Eu(W_1) &= \widehat{u}(\mu + \beta(y_{\max} - \mu) + \pi^f(y_{\max}) - (1 - \theta)P_1 + \theta T) \\ &= \widehat{u}(\mu + \beta(y_{\max} - \mu) + \pi^f(y_{\max}) - (1 - \theta)E(I_1(y)) + (1 - \theta)T - V_0 + \theta T) \\ &= \widehat{u}(\mu + \beta(y_{\max} - \mu) + \pi^f(y_{\max}) - (1 - \theta)E(I_1(y)) + T - V_0) \\ &= \widehat{u}(\mu + \beta(y_{\max} - \mu) + \pi^f(y_{\max}) - \beta(y_{\max} - \mu) - (\pi^f(y_{\max}) - E\pi^f(y)) + T - V_0) \\ &= \widehat{u}(\mu + E\pi^f(y) + T - V_0) \end{aligned}$$

which is exactly the same as  $Eu(W_0)$ . It is also straightforward to check that this same result holds true for the case  $\beta < 0$ . ■

## 5 Conclusion

Our primary finding is that farmers should be indifferent when considering the decision to purchase area-yield insurance from a private company or encompass that business in their cooperative. We derive this result for the specific case of costless insurance and assume a Pareto Optimal contract. Under these assumptions, the government subsidies that the cooperative would hope to capture are simply a net deduction in their premiums. In other words, the benefit they capture from the subsidies is the same when they purchase the insurance from an outside firm or internally. We also show that farmers will face greater risk when they internalize the insurance company, a factor ignored in previous empirical explorations of this issue. However, the cooperative is able to offset this increased risk by purchasing more insurance. If the government increased their subsidies so that they exceeded the premium deductions, the cooperative would clearly capture greater benefits and be able to pass these on to its members. Although theoretically this scenario is problematic, since the contracts would no longer be Pareto Optimal, in reality the idea that the government might consider such a policy as another vehicle for transferring funds to farmers is plausible.

Clearly, future research needs to test our findings in an environment with heterogenous agents, although our results provide some intuition. If the farmers in the cooperative are heterogenous (i.e., they are characterized by different  $\beta$ 's) they will offset each other's risk, but their welfare improvement from internally providing the insurance over the case where they purchase it from the private insurer might be minimal. If they were also characterized by different utility functions, then their welfare gains might increase. However, any significant welfare gains from the cooperative providing part of the insurance will most likely come from decreasing delivery and underwriting costs or if they are able to somehow create some scale economies from the additional business. Future research should also consider whether the cooperative would face decreased moral hazard problems.

In sum, although we explore the possibility of agricultural cooperatives selling government subsidized insurance in a relatively constrained and abstract situation, our results provide some insights into the viability of this concept and the benefits that farmers might obtain. Our work also provides the groundwork for additional analysis, which is warranted given the fact that cooperatives and farmers will continue to face production and throughput risk, the cost of the risks are greater in terms of firm survival, and the government is looking for strategies to increase farmer participation in their crop insurance programs.

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